

Math 211 - Bonus Exercise 11 (please discuss on Forum)

- 1) Prove that a finite nilpotent group G has a normal subgroup of any order dividing $|G|$. Hint: first prove the result for p -groups.
- 2) (Frattini's argument): if G is finite and $H \trianglelefteq G$ is a normal subgroup, let P be a Sylow p -subgroup of H . Show that G is generated by H and $N_G(P)$, i.e. $G = HN_G(P)$.
- 3) Let p be a prime number, and consider a subgroup $H \leq S_p$ such that the corresponding action $H \curvearrowright \{1, 2, \dots, p\}$ is transitive. Show that H must contain a p -cycle.